Math 3: Unit 1 Test
SAMPLE

100 points Name: _____

- **A photo ID should be out, on your desk.**
- **You will not be allowed to leave the room during the exam unless it is an emergency.**
- **Phones must be silent and put away. Any visible phone (smart watch, headphones, ipad etc.) will result in a grade F . Hands must remain in view during the exam.**
- **No graphing calculator.**
- **No credit will be given for solutions if clear work is not shown.**

On all problems containing exponents, express answer using only positive exponents

CIRCLE T FOR TRUE, F FOR FALSE. (2 points each)

T (F) (1) $\sqrt{81} = \pm 9$ radical notation mean just the positive root

(T) F (2) $\left(12a^7b\right)\left(-2a^{-2}b^{-8}\right) = \dfrac{-24a^5}{b^7}$.

T (F) (3) $\sqrt{a^2+b^4} = a+b^2$ Cannot square root terms individually

T (F) (4) Simplifying, $\dfrac{x^3+3x^2}{x^3+2} = \dfrac{x^3+3x^2}{x^3+2} = \dfrac{3x^2}{2}$ Terms don't divide out

(T) F (5) Dividing, $\dfrac{2+3i}{1-i} = -\dfrac{1}{2}+\dfrac{5}{2}i$ $\dfrac{2+3i}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{2+5i-3}{1-i^2} = \dfrac{-1+5i}{2}$

Fill in the blanks.

(6) Simplify completly: $\left(-2a^6b^{-1}\right)^4$ $\dfrac{16a^{24}}{b^4}$

(7) Simplify. $\dfrac{12x^5y^2}{18x^{-3}y^7}$ $\dfrac{2x^8}{3y^5}$

(8) Factor completely: x^3-64 $(x-4)(x^2+4x+16)$

(9) Multiply and simplify: $y^{-2/3}\left(y^{2/3}+5y^{8/3}\right)$ $y^0+5y^{6/3} = 1+5y^2$

(10) $16^{-3/4}$= $\dfrac{1}{8}$

$\dfrac{1}{16^{3/4}} = \left(\dfrac{1}{\sqrt[4]{16}}\right)^3 = \dfrac{1}{2^3}$

(11) Simplify: (4 points each)

(a) $\left(\dfrac{2x^{-3}y^{3/2}}{6x^2\sqrt{y}}\right)^{-1/3}$

$\left(\dfrac{y}{3x^5}\right)^{-1/3} = \left(\dfrac{3x^5}{y}\right)^{1/3}$

$= \dfrac{\sqrt[3]{3}\,x^{5/3}}{y^{1/3}}$

(b). $\left(\sqrt{x}+3y\right)^2 = (\sqrt{x}+3y)(\sqrt{x}+3y)$

$x+6y\sqrt{x}+9y^2$

(12) Factor Completely: (5 points each)

(a) $15x^2 - 7x - 2$

$(5x+1)(3x-2)$

(b) $20x^{-5/3} + 5x^{1/3} + 20x^{-2/3}$

factor out smallest power

$5x^{-5/3}(4 + x^2 + 4x^1)$

$5x^{-5/3}(x^2+4x+4)$

$\frac{5(x+2)^2}{x^{5/3}}$

to determine the power the the X's remaining, subtract power that was factored out from power that is there

$\frac{1}{3} - \frac{-5}{3}$

$= \frac{1}{3} + \frac{5}{3}$

$\frac{6}{3} = 2$

$-\frac{2}{3} - \frac{-5}{3}$

$-\frac{2}{3} + \frac{5}{3} = \frac{3}{3} = 1$

(c) $2x^4 - 162$

$2(x^4 - 81)$

$2(x^2-9)(x^2+9)$

$2(x-3)(x+3)(x^2+9)$

(d) $3x^2\sqrt{5x-1} + \frac{1}{2}x^3(5x-1)^{-1/2}(5)$

$x^2(5x-1)^{-1/2}(3(5x-1) + \frac{5}{2}x)$

$\frac{x^2(\frac{35x}{2} - 3)}{(5x-1)^{1/2}}$ OR

$\frac{x^2(35x-6)}{2(5x-1)^{1/2}}$

(13) Simplify: (6 points each)

(a) $\dfrac{1}{x+1}-\dfrac{2}{(x+1)^2}+\dfrac{3}{x^2-1}$

$= \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{(x-1)(x+1)}$ LCD $(x-1)(x+1)^2$

$= \frac{1}{x+1}\frac{(x-1)(x+1)}{(x-1)(x+1)} - \frac{2}{(x+1)^2}\frac{(x-1)}{(x-1)} + \frac{3}{(x-1)(x+1)}\frac{(x+1)}{x+1}$

$= \dfrac{(x-1)(x+1) - 2(x-1) + 3(x+1)}{(x-1)(x+1)^2}$

$\dfrac{x^2-1 \; : \; -2x+2+3x+3}{(x-1)(x+1)^2}$

$= \dfrac{x^2+x+4}{(x-1)^2(x+1)}$

(b) $\dfrac{\frac{1}{\sqrt{x+2}}-\frac{1}{\sqrt{x}}}{\frac{2}{\sqrt{x}}}$ $\frac{\sqrt{x+2}\sqrt{x}}{\sqrt{x+2}\sqrt{x}}$

(rationalize denom) $\frac{\sqrt{x+2}}{\sqrt{x+2}}$

$= \dfrac{\sqrt{x}-\sqrt{x+2}}{2\sqrt{x+2}}$

$= \dfrac{\sqrt{x}\sqrt{x+2}-(x+2)}{2(x+2)}$

(14) Solve. Express answer in interval notation. Show all work. No credit given for improper method. (6 points each)

(a) $|5x-3|<4$

"distance less than"

$-4 < 5x-3 < 4$

$-1 < 5x < 7$

$-\frac{1}{5} < x < \frac{7}{5}$

$\left(-\frac{1}{5}, \frac{7}{5}\right)$

(b) $3+10x-8x^2 \geq 0$

$(3-2x)(1+4x) \geq 0$

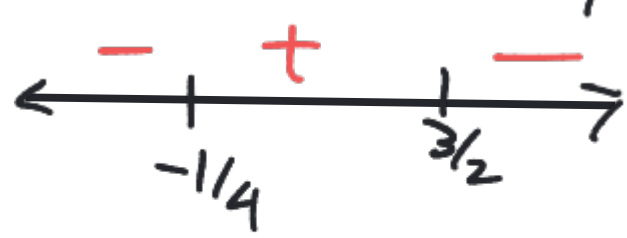
Consider equation

$(3-2x)(1+4x)=0$

$3-2x=0$ $\quad 1+4x=0$

$3=2x$ $\quad 4x=-1$

$x=3/2$ $\quad x=-1/4$



$[-1/4, 3/2]$

(15). Find all the solutions of the following equations (real or complex) (7 points each)

(a) $2+\sqrt{2x-1}=x$

$\sqrt{2x-1} = x-2$

* must check $\rightarrow (\sqrt{2x-1})^2 = (x-2)^2$

$2x-1 = x^2-4x+4$

$0 = x^2-6x+5$

$0 = (x-5)(x-1)$

$x=5$ ~~$x=1$~~ does not check

(b) $2x-3=2x^2$

$0 = 2x^2-2x+3$

$x = \frac{2\pm\sqrt{4-24}}{4}$

$= \frac{2\pm\sqrt{-20}}{4} = \frac{2\pm i\sqrt{20}}{4}$

$= \frac{2\pm i2\sqrt{5}}{4} = \frac{1\pm i\sqrt{5}}{2}$

(c) $\frac{3}{x}-\frac{2}{x-3}=\frac{-12}{x^2-9}$

$x(x+3)(x-3)\left(\frac{3}{x}-\frac{2}{x-3}\right) = \frac{-12}{(x+3)(x-3)}\left(x(x+3)(x-3)\right)$

$3(x+3)(x-3) - 2x(x+3) = -12x$

$3(x^2-9) - 2x^2-6x = -12x$

$3x^2-27-2x^2-6x+12x=0$

$x^2+6x-27=0$

$(x+9)(x-3)=0$

$x=-9,$ ~~3~~ does not check

(d) $x^4-5x^2+36=0$

Quadratic form

$u=x^2$

$u^2-5u+36=0$

$(u-9)(u+4)=0$

$u=9 \quad u=-4$

$x^2=9 \quad x^2=-4$

$x=\pm 3 \quad x=\pm\sqrt{-4}$

$=\pm 2i$